Small Numbers C&R

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Principal source: Wolfram MathWorld. The remaining content not specified in this document has been taken directly from Wolfram MathWorld and CODATA Recommended Values of the Fundamental Physical Constants.[1]

Hausdorff dimension of Blancmange curve

From The Takagi function: a survey.[2]

H. Dimension of Smith– Volterra–Cantor set

We start with the unit interval $[0, 1]$ and iteratively remove open middle intervals, but we remove geometrically smaller fractions at each step compared to the standard Cantor set.

To find the total length removed across all infinite steps, we sum the geometric series:

$$\sum_{n=1}^{\infty} 2^{n-1} \left(\frac{1}{4^n} \right) = \sum_{n=1}^{\infty} \frac{2^{n-1}}{2^{2n}} = \sum_{n=1}^{\infty} \frac{1}{2^{n+1}} = \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \dots$$

This is a geometric series with $a = 1/4$ and $r = 1/2$. Thus

$$\text{Total Removed} = \frac{1/4}{1 - 1/2} = \frac{1}{2}$$

Because we started with an interval of length 1 and removed a total length of $1/2$, the remaining set—the Smith–Volterra–Cantor set—has a Lebesgue measure of $1/2$.

For subsets of $\mathbb{R}$, the 1-dimensional Hausdorff measure $\mathcal{H}^1$ is exactly equal to the Lebesgue measure.

$$\mathcal{H}^1(S) = m(S) = \frac{1}{2}$$

[1] Mohr, P,. Newell, D., Taylor, B., & Tiesinga, E. (2024). CODATA Recommended Values of the Fundamental Physical Constants: 2022.

[2] Allaart, P., & Kawamura, K. (2011). The Takagi function: a survey.

The definition of Hausdorff dimension states that if the s-dimensional Hausdorff measure of a set is strictly greater than zero ($0 < \mathcal{H}^s(F) < \infty$), then its Hausdorff dimension is exactly s.

$$\mathcal{H}^1(S) = 1/2 > 0 \implies \dim_H \geq 1$$

Because S is a subset of the 1-dimensional real line $\mathbb{R} \implies \dim_H \leq 1$

$$\implies 1 \leq \dim_H(S) \leq 1$$

$$\therefore \dim_H(S) = 1$$

H. Dimension of Spectrum of Fibonacci Hamiltonian

From The Fractal Dimension of the Spectrum of the Fibonacci Hamiltonian[3]

H. Dimension of Cantor set

For geometric fractals that are perfectly self-similar, the Hausdorff dimension equals the similarity dimension.

The Cantor set C is constructed by taking an initial line segment and replacing it with $N = 2$ smaller copies of itself, each scaled down by a factor of $r = 1/3$. The entire set is perfectly composed of these two smaller identical copies.

The relationship between the number of pieces N, the scaling factor r, and the dimension s is given by the Moran equation:

$$Nr^s = 1$$

$$\implies 2\left(\frac{1}{3}\right)^s = 1 \implies 3^s = 2 \implies s = \frac{\log 2}{\log 3}$$

H. Dimension of Feigenbaum attractor

From Dimension of the Feigenbaum Attractor[4]

H. Dimension of Zeros of a Wiener process

Hausdorff dimension is always bounded above by Box dimension

We will find the upper bound, we look at the expected Box dimension of $\mathcal{Z} \cap [0, 1]$. We want to cover the unit time interval with N tiny intervals of length δ, and determine how many of them actually contain a zero.

[3]Damanik, D., Embree, M., Gorodetski, A., & Tcheremchantsev, S. (2008). The Fractal Dimension of the Spectrum of the Fibonacci Hamiltonian. Communications in Mathematical Physics, 280(2), 499–516.

[4]Pollicott, M. (2026). Dimension of the Feigenbaum Attractor.

Using the scaling property of Brownian motion, we have that W_{ct} has the same distribution as $\sqrt{c}W_t$.[5]

A consequence of Lévy's arcsine law is that the probability that a Wiener process has at least one zero in a small time interval $[t, t+\delta]$ is proportional to $\sqrt{\delta/t}$ (for $\delta \ll t$).

If we partition $[0,1]$ into $1/\delta$ intervals, the expected number of intervals $N(\delta)$ that intersect the zero set $\mathcal{Z}$ is the sum of these probabilities:

$$\mathbb{E}[N(\delta)] \approx \sum_{k=1}^{1/\delta} \sqrt{\frac{\delta}{k\delta}} = \sqrt{\delta} \sum_{k=1}^{1/\delta} \frac{1}{\sqrt{k}}$$

We know that $\sum_{k=1}^{n} 1/\sqrt{k} \approx 2\sqrt{n}$, so:

$$\mathbb{E}[N(\delta)] \approx \sqrt{\delta} \left(2\sqrt{\frac{1}{\delta}} \right) \approx C\delta^{-1/2}$$

Because the expected number of boxes of size δ needed to cover the zeros grows at a rate of $\delta^{-1/2}$. This strongly suggests—and in fact one can prove—that the Box dimension is $1/2$.

$$\therefore \dim_H(\mathcal{Z}) \leq 1/2$$

For the lower bound:

Local time measures the "amount of time" the Wiener process spends exactly at zero. It is defined as:

$$L_t = \lim_{\epsilon \to 0} \frac{1}{2\epsilon} \int_0^t \mathbf{1}_{(-\epsilon,\epsilon)}(W_s) ds$$

L_t increases when $W_t = 0$. Therefore, if we define a measure $\mu([a,b]) = L_b - L_a$, this measure is supported exactly and entirely on the zero set $\mathcal{Z}$.

A theorem of Lévy states that Brownian local time has the same distribution as the running maximum, and in fact local time satisfies the same Hölder estimates needed below.

for any small interval I of length δ, the accumulated mass (local time) is bounded:

$$\mu(I) \leq C\delta^{\alpha} \quad \text{for any } \alpha < 1/2$$

Because the mass in any δ-interval is bounded by δ^{α}, Frostman's Lemma guarantees that the Hausdorff dimension must be at least α. Since this holds for any $\alpha < 1/2$ we have that $\dim_H(\mathcal{Z}) \geq 1/2$.

$$\implies 1/2 \leq \dim_H(\mathcal{Z}) \leq 1/2$$

[5]Just the direct link c: == https://math.stackexchange.com/questions/644100/prove-the-scaling-property-of-a-brownian-motion

$$\therefore \dim_H(\mathcal{Z}) = 1/2$$

I realized that the Wikipedia article on the Wiener process already showed that its H dimension is $1/2$[6]

Volume of a unit ball in $\mathbb{R}^n$

Using the formula[7]

$$V_n(r) = \frac{r^n \pi^{n/2}}{\Gamma(\frac{n}{2}+1)}$$

For $n = 100$:

$$V_{100}(1) = \frac{\pi^{50}}{\Gamma(51)} = \frac{\pi^{50}}{50!} \approx 2.6 \times 10^{-40}$$

For $n = 500$

$$V_{500}(1) = \frac{\pi^{225}}{225!} \approx 5.7 \times 10^{-322}$$

I made a mistake in the video :c

For $n = 1000$

$$V_{1000}(1) = \frac{\pi^{500}}{500!}$$

For $n = 10^{100}$

$$V_{10^{100}}(1) = \frac{\pi^{5\times 10^{99}}}{(5 \times 10^{99})!}$$

Schwarzchild radius

I just used a S. radius calulcator.[8]

Probability of formation of a 300 amino acid protein

Assuming a system where the 20 standard proteinogenic amino acids are available in equal concentrations and bind completely at random, the number of possible combinations for a chain of 300 positions is $20^{300} \propto 10^{390}$.

Note that this pure-chance model is completely divorced from physical and biological reality, because it does not consider the following:

- Sequence Degeneracy (Redundancy)
- Evolution and abiogenesis do not require a 300-amino acid chain to pop into existence all at once. Natural selection operates cumulatively. Short, simple, randomly generated peptides (perhaps 10–20 amino acids long)

[6] https://en.wikipedia.org/wiki/Wiener_process

[7] Smith, D. J., & Vamanamurthy, M. K. (1989). How Small Is a Unit Ball? Mathematics Magazine, 62(2), 101–107.

[8] https://www.wolframalpha.com/input?i=schwarzschild+radius+calculator

that possess slight functional advantages are preserved, replicated, and combined over time.

- Amino acids do not bind purely at random. Their chemical properties (hydrophobicity, charge, size) dictate physical constraints, meaning certain sequences and folds are chemically favored over others, drastically narrowing the actual "search space" nature has to comb through.

Take the above probability with a grain of salt.

The probability that a true vacuum bubble will spontaneously nucleate

By the Tunneling Probability Formula

$$\Gamma \propto e^{-B/\hbar}$$

With $S_B \approx \frac{8\pi^2}{3|\lambda_{min}|}$,[9] the value of λ_{min} for high energies is around of -0.013.

$$\implies S_B \approx 2000$$

So

$$\Gamma \approx e^{-2000}$$

Probability of a n kg black hole materializing due to vacuum fluctuations

A simplification based on Bekenstein–Hawking entropy.

Probability that a monkey on a typewriter will type all the texts written by humanity

Google estimated there were roughly 130 million unique books in 2010.
Let's assume 200 million books today. Assuming average book has about 500 000 characters. That's 10^{14} characters just for published books.
Adding all emails, text messages, scientific papers, codebases, forums, and ancient scrolls, the estimation is around 10^{20} characters.

$$\implies P = \left(\frac{1}{50}\right)^{10^{20}} \approx 10^{-2\times 10^{16}}$$

[9]Isidori, G., Rychkov, V. S., Strumia, A., & Tetradis, N. (2008). Gravitational corrections to standard model vacuum decay. Physical Review D—Particles, Fields, Gravitation, and Cosmology, 77(2), 025034.

De Sitter recurrence

yep[10]

Recurrence of the observable universe, Recurrence of the inflationary multiverse (Linde & Page)

From this source[11]

Okojo-ermine number

https://googology.fandom.com/wiki/Okojo_numbers

Infinitesimals

See Wikipedia #infinity section[12]

[10]K. Ropotenko. (2007). The Poincare recurrence time for the de Sitter space with dynamical chaos.
[11]Don N. Page. (1994). Information Loss in Black Holes and/or Conscious Beings?.
[12]https://en.wikipedia.org/wiki/Surreal_number#Infinity